VIVIANE BALADI
CNRS/Ecole Normale Supérieure

Exponential decay of correlations for piecewise hyperbolic contact flows

Strong ergodic properties (such as exponential mixing) have been proved for various smooth dynamical systems by first obtaining a spectral gap for a suitable "transfer" operator acting on an appropriate Banach space. Some natural dynamical systems, such as discrete or continuous-time billiards, are only piecewise smooth, and the discontinuities pose serious technical problems in the definition of the Banach norm. In the discrete-time situation, with Sebastien Gouezel (J Mod Dyn 2010), we overcame these problems by using classical tools such as complex interpolation on anisotropic Sobolev Triebel spaces, and an old result of Strichartz on Fourier multipliers. With Carlangelo Liverani, we now obtain exponential decay of correlation for piecewise hyperbolic contact flows. This is possible by combining the ideas in the work with Gouezel on discrete-time dynamics with techniques developed by Liverani a few years ago (based on ideas introduced by Dima Dolgopyat) to study contact Anosov flows.

LUIS BENET
Universidad Nacional Autónoma de México

Escaping orbits shaping narrow planetary rings: A billiard example

SERGEY BOLOTIN
University of Wisconsin-Madison

Second species solutions of the 3 body problem

We consider plane 3 body problem with two of the masses small. Second species solutions of Poincaré are periodic solutions with near collisions of small masses. After fixing total energy, description of almost collision solutions is reduced to dynamics of a discrete Lagrangian system with 2 degrees of freedom whose Lagrangian is defined in terms of Lambert's problem. Reducing rotational symmetry gives a discrete Lagrangian system with 1 degree of freedom. In certain cases dynamics of this system can be analyzed which gives many second species solutions.

FLORENTINO BORONDO
Universidad Autonoma de Madrid

Quantum-like Poincare-Birkhoff theorem

Poincare's pioneering work made at the dawn of the XX century on the three body problem with the occasion of the mathematical prize honoring King Oscar II 60th birthday [1] is remarkable in many aspect. For one thing, it demonstrated the possibility of irregular motion, opening (very early) the door for the development of Chaos Theory that took place in the 1960's gathering master contributions by Lorenz (Kyoto Prize), Mandelbrot and others.

Moreover, the importance of Poincare's superb work does not end there, since he also demonstrated the existence of a definite structure in the complexity of chaos, where periodic orbit, and its associated homoclinic and heteroclinic orbits play the major role. The loose ends of Poincare's original work where completed also in the 1960's with the celebrated Poincare-Birkhoff (PB) and Kolmogorov-Arnold-Moser (KAM) theorems.

These results provide, together with Arnold diffusion a complete view and full understanding of the hierarchical organization of the complexity of classical Hamiltonian chaos. Another relevant point in this story is that shortly after Poincare's work was done, quantum theory was born, pointing out the limitations of the classical theory, and then of his results. The correspondence between these two mechanics then became an active field of research, problem (often referred to as Quantum Chaos) which is still largely open in the case of chaotic systems.

Nevertheless, substantial contributions to this field have been made (often published in PRL), such as Gutzwiller trace formula [2], random matrix theory [3], or scar theory [4], just to name a few most related to our work. In all of them, the importance of classical invariants in the quantum theory is clearly shown, but regretably very few papers directly addressed which is the role, if any, of the two capital theorems referenced above at quantum level. In this respect, the results of Geisel et al. [5] showing the quantum effect of cantori (important in KAM theory) the quantum version of KAM theorem discussed by Taylor et al [6], and quantum Arnold difussion [7] are worth mentioning.

In this Letter, we also progress along this line, by presenting uncontroversial numerical evidence that the structures implied in the PB theorem clearly appear at quantum level, also disentangling the underlying mechanism. We truly believe that contributions like ours can help in the near future to transfer the detailed understanding of chaos at classical level provided by Poincare to the quantum world, making a significant step towards closing the still open problem of quantum chaos.

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ABED BOUNEMOURA
Institut Mathématique de Jussieu

Stability and instability for near-linear Hamiltonian systems

We shall consider perturbations of linear integrable Hamiltonians systems (i.e. with a constant frequency). Assuming the frequency is non-resonant, we shall first prove a non-trivial result of effective stability, and then construct an example of instability showing that both the result and the example are the best possible.

HENK BROER
University of Groningen

Resonance and fractal geometry

The phenomenon of resonance will be dealt with from the viewpoint of dynamical systems depending on parameters and their bifurcations. Resonance phenomena are associated to open subsets in the parameter space, while their complement corresponds to quasi-periodicity and chaos. The latter phenomena occur for parameter values in fractal sets of positive measure. We describe a universal phenomenon that plays an important role in modelling.

LEONID BUNIMOVICH
Georgia Institute of Technology

Focusing components of chaotic billiards should be absolutely focusing

It is known that mechanisms of chaos (hyperbolicity) in billiards are dispersing and defocusing. The same mechanisms work for geodesic flows. Dispersing works when all boundary components are dispersing and this mechanism does not require any other conditions on a structure of a billiard table. On another hand, there are chaotic as well as integrable billiards when the boundary of a billiard table contains focusing components. therefore a natural question is which focusing components (curves) can belong to the boundary of a chaotic and ergodic billiard. We show that essentially a necessary condition is that all focusing components must be absolutely focusing ones. More precisely, a non-absolutely focusing component can not be "defocused" by moving all other components of the boundary arbitrarily far away from it. (joint work with A.Grigo)

WEI CHENG
Nanjing University

Integrability by means of variational methods

NIKOLAI CHERNOV
University of Alabama at Birmingham

Stable regimes for hard disks in a channel with twisting walls

A Hamiltonian system of N hard disks in a strip bounded by two rigid walls is expected to be hyperbolic, ergodic, and mixing (this is all proven for $N = 2$). One can modify the collisions at the walls in order to generate a shear flow in the channel. It is expected that the modified systems has a non-equilibrium steady state (NESS) that is a Sinai-Ruelle-Bowen (SRB) measure. We show, however, that if the disks are too small, then for any modification of the wall collisions the dynamics tends to collapse and degenerate to simple periodic oscillations.

LUIGI CHIERCHIA
Universita "Roma Tre"

Full torsion in the planetary N-body problem

Thanks to symplectic variables recently introduced, full torsion (or twist) in the general planetary N-body problem can be proved. This allows to "complete" the KAM analysis for the planetary problem and to draw several dynamical consequences (for small planetary masses, in the nearly co-planar and co-circular regime): existence and relatively large measure of non-degenerate Kolmogorov Lagrangian tori, Birkhoff-Lewis periodic orbit cumulating on the Kolmogorov set, lower dimensional elliptic tori. On the other hand the problem of exponential stability of "adiabatic variables" in open sets is essentially open and I will briefly discuss what is known and what one would like to know.

PABLO CINCOTTA
Universidad Nacional de La Plata

Open problems in Dynamical Astronomy

The aim of the present talk is to discuss and present some unsolved problems in galactic and asteroidal dynamics. In the first part the problem of constructing equilibrium models of galaxies is considered, pointing out several difficulties concerning diffusion in phase space of multidimensional dynamical system in physical times. The Jeans theorem is fully discussed as well as its generalization due to Merritt and Efthimiopoulos. In the second part, I plan to present an application of the standard Chirikov's diffusion theory to asteroidal dynamics in a three body resonance, that seems to work even though the model does not fulfill all the necessary hypothesis of this theory.

JACOPO DE SIMOI
University of Roma "Tor Vergata"

Cyclicity one elliptic islands in the standard family

We study abundance of a special class of elliptic islands for the standard family of area preserving diffeomorphism for large parameter values, i.e. far from the KAM regime. Outside a bounded set of parameter values, we prove that the measure of the set of parameter values for which an infinite number of such elliptic islands coexist is zero. On the other hand we construct a positive Hausdorff dimension set of arbitrarily large parameter values for which the associated standard map admits infinitely many elliptic islands whose centers accumulate on a locally maximal hyperbolic set.

AMADEU DELSHAMS
Universitat Politécnica de Catalunya

A geometric mechanism of diffusion in a priori unstable Hamiltonian systems

We consider a priori unstable Hamiltonian systems which are (quasi) periodic perturbations of one (or some) pendulum plus one (or some) rotor. All these models present the so called large gap problem. We apply to these systems a geometric mechanism for diffusion, and provide explicit and easily verifiable conditions for the existence of diffusing orbits for the rotor variables. In particular, we describe the construction of a scattering map and the combination of two types of dynamics on a very large invariant object called NHIM (normally hyperbolic invariant manifold).

This talk is based on joint work with Rafael de la Llave and Tere M. Seara.

DMITRY DOLGOPYAT
University of Maryland

Instabilities of area preserving flows on surfaces

We consider small dissipative perturbations of area preserving flows on surfaces. We show that a viscosity type regularization of such systems leads to a Markov process on the space of ergodic components of the unperturbed system. In the higher genus case this can give rise to an intermitent behaviour. This is a joint work with Mark Freidlin and Leonid Koralov.

LARA EL SABBAGH
Université Paris VI

Inclination lemmas for normally hyperbolic manifolds with applications to diffusion

The notion of instability in near-integrable Hamiltonian systems goes back to Arnold. In his famous note (1964) he gave an example of a three-degree-of-freedom system where unstable orbits shadowing whiskered tori were constructed. The mechanism is based on the existence of a transition chain, that is, a family of invariant tori with heteroclinic connections. He gets the orbit connecting the extremal tori of this chain (the unstable orbit) by an obstruction argument adapted to partially hyperbolic tori.

In this talk, we state two λ-lemmas for normally hyperbolic invariant manifolds which will turn out to be a tool for proving a weaker obstruction argument and thus finding diffusion orbits in more general cases.

GERARD GOMEZ
Universitat de Barcelona

Some Transport Mechanisms in the Solar System

In this talk we will analyze two possible mass transport mechanisms in the Solar System. One of them is related to the delivery of Mars ejecta to Earth and the other to transport from the outer to the inner solar system. The study is based in the analysis of the invariant stable and unstable manifolds associated to some relevant periodic orbits.

MARCEL GUARDIA
Fields Institute/ U. of Toronto

Diffusion along mean motion resonances in the restricted three body problem

We consider the 3 Body Problem for the Sun, Jupiter and an Asteroid, which we assume that has negligible mass. Taking a realistic value of the mass ratio and the eccentricity of the primaries arbitrarily small, we study the regime of the mean motion resonance 1:7, namely when the period of the Asteroid is approximately seven times the period of Jupiter. It is well known that if one neglects the influence of Jupiter on the Asteroid, the orbit of the latter is an ellipse. In this talk we will show how the influence of Jupiter causes substantial changes on the shape of Asteroid's orbit. This is a joint work with J. Fjoz, V. Kaloshin and P. Roldan.

MASSIMILIANO GUZZO
University of Padova

Numerical investigations of a conjecture by N.N. Nekhoroshev about stability in quasi-integrable systems

We discuss numerical experiments about a conjecture by N.N. Nekhoroshev on the influence of a geometric property, called steepness, on the long term stability of quasi-integrable systems. In a Nekhoroshev's 1977 paper, it was conjectured that, among the steep systems with the same number of frequencies, the convex ones are the most stable, and it was suggested to investigate numerically the problem. Following this suggestion, we numerically study and compare the diffusion of the actions in quasi-integrable systems with different steepness properties in a large range of variation of the perturbation parameter.

JEAN-PIERRE MARCO
Université Paris 6

Generic properties of classical systems on $\mathbb{T}^2$ with applications to diffusion.

We consider classical systems on $\mathbb{A}^2 = T^*\mathbb{T}^2$ of the form

$$C(\theta, r) = \frac{1}{2}T(r) + U(\theta)$$

where T is a positive definite quadratic form on $\mathbb{R}^2$ and U a potential function in $C^\kappa(\mathbb{T}^2)$ for κ large enough. Under generic conditions, such a system admits a hyperbolic fixed point for some critical energy, together with homoclinic orbits.

We will give a set of (generic) nondegeneracy conditions under which, for each nonzero integer homology class c on $\mathbb{T}^2$, we can prove the existence of chains of hyperbolic cylinders realizing c. These cylinders are families of hyperbolic orbits, parametrized by the energy, whose projections on $\mathbb{T}^2$ belong to the class c. The chains are finite families of such cylinders, with heteroclinic connections between them, going from the critical energy to the infinite energy. At the critical energy, the "boundary" of the asymptotic cylinder is a concatenation of homoclinic orbits of the fixed point.

By means of a suitable version of the Birkhoff-Smale theorem, we can then prove the existence of two complementary subsets $\hat{\mathcal{H}}$ and $\check{\mathcal{H}}$ of $H_1(\mathbb{T}^2, \mathbb{Z}) \setminus \{0\}$, with $\hat{\mathcal{H}} = -\check{\mathcal{H}}$, such that when c and c' are in the same subset, the associated chains admit heteroclinic connections.

We will finally introduce the notion of singular cylinder, in order to connect chains of cylinders realizing opposite homology classes.

Classical systems are natural normal forms for a priori stable three-degrees-of-freedom systems at double resonances. As an application of the previous study, we will show how to exhibit a normally hyperbolic skeleton for Arnold diffusion in a priori stable systems, whose projection on the action space tends to be dense in the energy level when the perturbation tends to 0.

JOHN MATHER
Princeton University

Near Double Resonance

I will describe the location and properties of the Aubry sets near a strong double resonance in my proof of Arnold Diffusion.

P. J. MORRISON
University of Texas at Austin

Transport, Arnold Diffusion, Stability, and Negative Energy Modes

A sequence of physical models, ranging from partial differential equations to low-dimensional symplectic maps, will be used to illustrate the importance of transport in Hamiltonian systems with more than two degrees of freedom. The models will be drawn from the physics of fluids and plasmas. General features of a large class of models of continuum physics will be noted.

ANATOLY NEISHTADT
Loughborough University/Space Research Institute

Averaging, passages through resonances, and captures into resonance in dynamics of charged particles

A study of motion of charged particles in the electromagnetic field of a wave propagating in plasma in the presence of a constant background magnetic field is a classical problem in plasma physics. Under different relations between parameters of this problem completely different dynamical phenomena take place. The averaging method is a natural tool for study of this problem. In this talk I am planning to describe phenomena of capture into resonance and scattering on resonance that occur in the case of slow high frequency waves and a weak background magnetic field. In this case particle motion is described by a slow-fast Hamiltonian system. Capture into resonance may lead to unlimited particle acceleration along the front of the wave (so called surfatron acceleration).

MARIA SAPRYKINA
KTH School of Engineering Sciences

Examples of Hamiltonian systems with Arnold diffusion

I shall speak about new results obtained in collaboration with Vadim Kaloshin and Mark Levi.

Here is a heuristic description of the problem. Imagine $n \geq 4$ mathematical penduli attached to the wall in a line and moving. If they are disjoint, the energy of each pendulum is preserved for all time. Now we join each pair of neighboring penduli by a thin rubber band. Of course, the total energy of the system is still preserved. But what happens with the energy of each individual pendulum? KAM theorem asserts that under some generic assumptions, for "most" initial conditions the energy of each pendulum will stay close to the initial one for all time. One of our results asserts that one can find such initial conditions and such moments of time t_j, that at time t_j the j-th pendulum moves with almost the total energy of the system.

Another result is the following. For any r we construct a C^∞–Hamiltonian, which is C^r-close to $H_0(I) = \dfrac{\langle I, I \rangle}{2}$ and has a trajectory dense in a set of maximal Hausdorff dimension on the unit energy surface.

DAVID SAUZIN
Observatoire de Paris

There is only one KAM curve

We consider the standard family of area-preserving twist maps of the annulus and the corresponding KAM curves. Inspired by Kolmogorov, Arnold and Herman, we show that, instead of viewing these invariant curves as separate objects, each of which having its own Diophantine frequency, one can encode them in a single function of the frequency which is naturally defined in a complex domain containing the real Diophantine frequencies and which is monogenic in the sense of Borel; this implies a remarkable property of quasianalyticity, although real frequencies consitute a natural boundary for analytic continuation from the Weierstrass point of view because of the density of the resonances.

ALFONSO SORRENTINO
University of Cambridge

On the existence of invariant smooth Lagrangian graphs: a weak version of Liouville-Arnol'd theorem

In the study of Hamiltonian systems, a special role is played by invariant Lagrangian submanifolds. These objects arise quite naturally in many physical and geometric problems and share a deep relation with the dynamics of the system and with the Hamiltonian itself.

In this talk I'll address the question of the existence of invariant smooth Lagrangian graphs in the case of Tonelli Hamiltonians with "sufficiently" many independent integrals of motions, but not necessarily in involution. I'll discuss some results that are reminiscent of Liouville-Arnol'd theorem and, quite surprisingly, prove that this (a-priori) weaker notion of integrability implies the classical one in the case of the n-dimensional torus. Time permitting, I'll also discuss analogous results in the case of left-invariant Tonelli Hamiltonians on exponential-type sol- manifolds. The main difference in this case, is that these systems possess symmetries that do not come from conserved quantities, but are related to the action of an amenable group on the universal cover of the manifold. This latter part is a joint work with Leo Butler.

ARTURO VIEIRO
Universitat de Barcelona

Dynamics in resonant chaotic zones: inner/outer splittings of separatrices

Let F_ϵ be a one-parameter family of area-preserving maps (APMs) having an elliptic fixed point E_0. Typically, when changing ϵ, different chains of resonant Birkhoff islands of stability $I_{p/q}$ bifurcate from E_0. Generically, these stability islands have a pendulum-like phase space structure. Hence, for a given island $I_{p/q}$ one should consider two "main" splittings of separatrices, geometrically related to the upper and lower separatrices of the classical pendulum. We shall refer to these splittings as the inner/outer splitting, according to their distance to E_0. They are generically different, the outer being the largest one (for islands located close enough to E_0).

We will comment on the use of the Chirikov separatrix map to quantitatively describe the dynamics within the chaotic region that the inner/outer splittings create around $I_{p/q}$. In particular, we will describe the consequences that produce the difference between these splittings on the shape and size of the chaotic region. Some examples will illustrate the theoretical results on this point.

Some of the related results can be found in

C. Simó and A. Vieiro. *Dynamics in chaotic zones of area preserving maps: close to separatrix and global instability zones*. Phys. D 240(8), 2011.

JINXIN XUE
University of Maryland

Continuous averaging proof of the Nekhoroshev theorem

JUN YAN
Fudan University

A New Lax-Oleinik Type Operator in Weak KAM Theory

KE ZHANG
University of Toronto

Arnold diffusion via normally hyperbolic cylinders

This is joint work with P.Bernard and V.Kaloshin. We prove a version of Arnold diffusion for a priori stable Hamiltonian systems. We will show that there exists normally hyperbolic cylinders for the system near a single resonance, and the size of the cylinder does not decrease to zero as the perturbation goes to zero. We will also discuss some ongoing work with V.Kaloshin, which deals with similar objects near a double resonance.