Dangerous Knowledge:

Credit Value Adjustment with Credit Triggers

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Disclaimer

- Opinions expressed in this talk are those of the author and do not reflect any views or policies of RBC;
- Dangerous Knowledge Documentary: "...looks at four brilliant mathematicians - Georg Cantor, Ludwig Boltzmann, Kurt Godel and Alan Turing - whose genius has profoundly affected us, but which tragically drove them insane and eventually led to them all committing suicide."

–from Docuwiki

Outline

Notations

- Names:
  - Name 0: investor (Ex. CDS protection buyer);
  - Name 1: reference entity (Ex. CDS reference entity);
  - Name 2: counterparty (Ex. CDS protection seller);
- Default Times: $\tau_i, \quad i = 0, 1, 2;$
- Constant Loss Given Default: $l_i, \quad i = 0, 1, 2;$
- Probability Space: $(\Omega, \mathcal{G}, \mathcal{G}_t, \mathbb{Q})$
  - Total Information: $\mathcal{G}_t := \mathcal{F}_t \vee \mathcal{H}_t;$
  - Market Observable Quantities: $\mathcal{F}_t;$
  - Default Filtration: $\mathcal{H}_t := \sigma(\{\tau_0 \leq u \vee \tau_1 \leq u \vee \tau_2 \leq u : u \leq t\});$
- Cash Flows: $\Pi_t^2(T_1, T_2) = -\Pi_t^0(T_1, T_2)$
  - $\Pi_t^0(T_1, T_2)$: investor's discounted cash flows subjected to cpty default;
  - $\Pi_t(T_1, T_2)$: investor's analogous quantity when the cpty is default-free;
  - $\Pi_t^2(T_1, T_2)$: cpty's discounted cash flows subjected to investor default;
  - Discounted Future MtM: $A_g := \mathbb{E}_g[\Pi_g(g, T)]D(t, g), \quad g \in (t, T].$

General Unilateral Counterparty Risk Pricing Formula

Proposition

Assuming the investor is default-free, given $\tau_2 > t$, the price of the investor's payoff under unilateral cpty risk (Mark-to-Credit) is:

$$\mathbb{E}_t[\Pi_t^0(t, T)] = \mathbb{E}_t[\Pi_t(t, T)] - CVA_0; \quad CVA_0 = l_2 \mathbb{E}_t[1_{\{\tau_2 \leq T\}} A_{\tau_2}^+];$$

where CVA_0 is the investor's unilateral credit value adjustment. Assuming the cpty is default-free, given $\tau_0 > t$, the price of the cpty's payoff under unilateral cpty risk is:

$$\mathbb{E}_t[\Pi_t^2(t, T)] = \mathbb{E}_t[-\Pi_t(t, T)] - CVA_2; \quad CVA_2 = l_0 \mathbb{E}_t[1_{\{\tau_0 \leq T\}} (-A_{\tau_0})^+];$$

where CVA_2 is the cpty's unilateral credit value adjustment.

General Bilateral Counterparty Risk Pricing Formula

Proposition

Let $\tau = \tau_0 \wedge \tau_2$. Given $\tau > t$, the price of the investor's payoff under bilateral cpty risk is:

$$\begin{aligned} \mathbb{E}_t[\Pi_t^0(t,T)] &= \mathbb{E}_t[\Pi_t(t,T)] - BCVA_0; \\ BCVA_0 &= l_2 \mathbb{E}_t[1_{\{\tau=\tau_2 \le T\}} A_{\tau_2}^+] - l_0 \mathbb{E}_t[1_{\{\tau=\tau_0 \le T\}} (-A_{\tau_0})^+]; \end{aligned}$$

where $BCVA_0$ is the investor's bilateral credit value adjustment;
Given $\tau > t$, the price of the cpty's payoff under bilateral cpty risk is:

$$\mathbb{E}_t[\Pi_t^2(t,T)] = \mathbb{E}_t[-\Pi_t(t,T)] - BCVA_2;$$

where $BCVA_2 = -BCVA_0$ is the cpty's bilateral credit value adjustment.

Symmetry vs Asymmetry

- CVA: defined as the negated diff of the value of a cpty-defaultable cash flows and the value of an analogous cpty-default-free cash flows;
- Asymmetry of unilateral CVA: $0 \leq CVA_0 \neq -CVA_2 \leq 0$;
  - Asymmetry of the unilateral CVA is due to asymmetric assumptions;
  - CVA_0 computes the investor's expected loss due to cpty default given no previous cpty default, assuming the investor is default-free;
  - CVA_2 computes the cpty's expected loss due to investor default given no previous investor default, assuming the cpty is default-free;
- Symmetry of bilateral CVA: $BCVA_2 = -BCVA_0$;
  - The BCVA_0 is composed of the expected loss due to the cpty defaults (before the investor default) minus the gain made by the investor in the event of its own default (before the cpty default), given no previous default for both parties;
  - Symmetry is due to the same consideration that both parties are risky;

Credit Triggers

- Credit Triggers: a pre-specified credit level that is below the current rating of the cpty. When the cpty's credit deteriorates to its credit trigger or below before default within maturity, the investor has the right to settle the deal with its cpty;
- Motivation: monitor/mitigate cpty credit risk;
- Obliged Credit Triggers: obliged to settle at credit trigger event;
- Optional Credit Triggers: optional to settle at credit trigger event;
- Credit Trigger Times: τ_i^b

$$
\begin{aligned}
1 &= P(\tau_i^b \leq \tau_i) = P(\tau_i^b < \tau_i) + P(\tau_i^b = \tau_i) \\
\tau^b &:= \tau_0^b \wedge \tau_2^b;
\end{aligned}
$$

CVA with Obliged Credit Triggers

Proposition

Given $\tau_2^b > t$ (given $\tau_0^b > t$), the unilateral CVA with obliged credit trigger CVA_0^b (CVA_2^b) for the investor (cpty) is given by:

$$CVA_0^b = l_2\mathbb{E}_t[1_{\{\tau_2^b=\tau_2\leq T\}}A_{\tau_2}^+]; \quad CVA_2^b = l_0\mathbb{E}_t[1_{\{\tau_0^b=\tau_0\leq T\}}(-A_{\tau_0})^+].$$

Given $\tau^b > t$, the bilateral CVA with obliged credit triggers of the investor $BCVA_0^b$ (cpty: $BCVA_2^b$) is given by:

$$\begin{aligned} BCVA_0^b &= l_2\mathbb{E}_t[1_{\{\tau_0\geq\tau_2=\tau^b\leq T\}}A_{\tau_2}^+] - l_0\mathbb{E}_t[1_{\{\tau_2\geq\tau_0=\tau^b\leq T\}}(-A_{\tau_0})^+]; \\ BCVA_2^b &= l_0\mathbb{E}_t[1_{\{\tau_2\geq\tau_0=\tau^b\leq T\}}(-A_{\tau_0})^+] - l_2\mathbb{E}_t[1_{\{\tau_0\geq\tau_2=\tau^b\leq T\}}A_{\tau_2}^+]. \end{aligned}$$

CVA with Optional Credit Triggers: Unilateral

- Discounted Future Mark-to-Credit (MtC):

$$A_g^i = \mathbb{E}_g[\Pi_g^i(g, T)]D(t, g); \quad g \in (t, T].$$

Proposition

Given $\tau_2^b > t$ (given $\tau_0^b > t$), the unilateral CVA with optional credit triggers $\overline{CVA}_0^b$ ($\overline{CVA}_2^b$) for the investor (cpty) is given by:

$$\begin{aligned}
\overline{CVA}_0^b &= l_2 \mathbb{E}_t[(1_{\{\tau_2^b = \tau_2 \le T\}} + 1_{\{\tau_2^b < \tau_2 \le T, A_{\tau_2^b} < A^0_{\tau_2^b}\}}) A_{\tau_2}^+]; \\
\overline{CVA}_2^b &= l_0 \mathbb{E}_t[(1_{\{\tau_0^b = \tau_0 \le T\}} + 1_{\{\tau_0^b < \tau_0 \le T, -A_{\tau_0^b} < A^2_{\tau_0^b}\}})(-A_{\tau_0})^+].
\end{aligned}$$

CVA with Optional Credit Triggers: Bilateral

Proposition

Given $\tau^b > t$, the bilateral CVA with optional credit triggers $\overline{BCVA}_0^b$ ($\overline{BCVA}_2^b$) for the investor (cpty) is given by:

$$
\begin{aligned}
\overline{BCVA}_0^b \;=\;& l_2\mathbb{E}_t[1_{\{\tau_0 \geq \tau_2 = \tau^b \leq T\}} A^+_{\tau_2}] + l_2\mathbb{E}_t[1_{\{\tau_0^b > \tau_2^b < \tau_2 = \tau \leq T, A_{\tau_2^b} < A^0_{\tau_2^b}\}} A^+_{\tau_2}] \\
& + l_2\mathbb{E}_t[1_{\{\tau_2^b > \tau_0^b < \tau_2 = \tau \leq T, -A_{\tau_0^b} < A^2_{\tau_0^b}\}} A^+_{\tau_2}] \\
& - l_0\mathbb{E}_t[1_{\{\tau_2 \geq \tau_0 = \tau^b \leq T\}} (-A_{\tau_0})^+] \\
& - l_0\mathbb{E}_t[(1_{\{\tau_0^b > \tau_2^b < \tau_0 = \tau \leq T, A_{\tau_2^b} < A^0_{\tau_2^b}\}})(-A_{\tau_0})^+] \\
& - l_0\mathbb{E}_t[1_{\{\tau_2^b > \tau_0^b < \tau_0 = \tau \leq T, -A_{\tau_0^b} < A^2_{\tau_0^b}\}} (-A_{\tau_0})^+] \\
\overline{BCVA}_2^b \;=\;& -\overline{BCVA}_0^b.
\end{aligned}
$$

Model Risks

- Model Choices: Pure Diffusion, Pure Jumps or Jump Diffusion? Affect jump-to-default probabilities: $P(\tau_i^b = \tau_i \leq T)$;
- Jump Size Distributions: Two different models with same default probability term structure may produce different jump-to-default probabilities;
- Calibration Issues: lack of data for jump-to-default probabilities; only data available is total default probabilities;

Numerical Example: Unilateral Obliged

- Compound Poisson Model:

$$X_t = \sum_{i=1}^{N_t} Y_i + x_0; \quad x_0 > b > 0;$$
$$\tau = \inf\{t \geq 0 : X_t \leq 0\}; \quad \tau^b = \inf\{t \geq 0 : X_t \leq b\};$$

  where $N_t \sim Poisson(\lambda)$ and $Y_1, Y_2, ...$ are i.i.d.
- Normal Jump (NJ) Model: Y_i has Normal distribution;
- Negative Exponential Jump (NEJ) Model: $-Y_i$ has Exp distribution;
- Trigger Ratio: $P(\tau^b = \tau \leq T)/P(\tau \leq T)$;

Trigger Ratios: NJ vs NEJ

$T = 1$	p^b	2.89%	6.12%	11.12%
Credit Barrier	NJ	0.5000	1.0000	1.5000
	NEJ	2.0171	4.0000	5.5816
Trigger Ratios	NJ	89.26%	77.32%	68.39%
	NEJ	95.23%	90.70%	87.19%
$T = 5$	p^b	15.67%	25.61%	38.76%
Credit Barrier	NJ	0.5000	1.0000	1.5000
	NEJ	2.0354	3.6850	5.098
Trigger Ratios	NJ	72.90%	48.01%	31.63%
	NEJ	81.58%	68.61%	58.73%
$T = 10$	p^b	29.06%	40.96%	54.94%
Credit Barrier	NJ	0.5000	1.0000	1.5000
	NEJ	1.5971	3.1055	4.4574
Trigger Ratios	NJ	64.05%	36.55%	20.26%
	NEJ	76.73%	58.91%	45.84%

Conclusions

- Generalized CVA formulae with credit triggers;
- Unilateral CVAs are reduced with credit triggers;
- Increased bilateral CVA due to credit triggers for one party is exactly the same amount of bilateral CVA reduced for the other party;
- CVA with credit triggers is subjected to large uncertainty of model risks, mostly due to the lack of data for calibrating jump-to-default probabilities;

Prior Credit Crisis...

"The most dangerous part is when people believe everything coming out of the model."

David X. Li

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