THE FIELDS INSTITUTE SEMINARS

ARITHMETIC AND GEOMETRY OF HIGHER DIMENSIONAL VARIETIES

WITH SPECIAL EMPHASIS ON

CALABI–YAU VARIETIES AND MIRROR SYMMETRY

ABSTRACTS
November 8, 2003

10:00am: Ken Ono (University of Wisconsin, Madison)

The trace formula and the modularity of a rigid Calabi–Yau threefold

We establish a simple inductive formula for the trace of the p-th Hecke operator on the space $S_k^{\text{new}}(\Gamma_0(8))$ of newforms of level 8 and weight k in terms of the values of ${}_3F_2$-hypergeometric functions over the finite field $\mathbf{F}_p$. Using this formula when $k = 6$, we prove a conjecture of Koike relating to the values ${}_6F_5(1)$ and ${}_4F_3(1)$. These formulas depend on the modularity of Calabi-Yau threefolds. In fact this shows how the modularity of a single threefold can dictate the Hecke actions on all spaces of modular forms on $\Gamma_0(8)$. Furthermore, we find new Beukers type congruences between traces of Hecke operators and Apery numbers.

11:00am: Ling Long (Iowa State University)

On cusp forms of non-congruence subgroups

Theory of modular form of noncongruence subgroups is far less developed compared to theory of congruence modular form. The Atkin and Swinnerton-Dyer conjecture is one of the major conjectures for cusp forms of noncongruence subgroups. While there has been significant progress, for example, the weak Atkin-Swinnerton-Dyer conjecture proved by Scholl, the full version of the conjecture still remain to be intractable. I plan to discuss a particular situation where some spaces of cusp forms of nongongruence subgroups admit bases satisfying the Atkin and Swinnerton-Dyer congruence relations with certain newforms of congruence subgroups. Consequently, the Atkin and Swinnerton-Dyer conjecture for these spaces are established.

12:00: Mike Roth (Queen's University)

The affine stratification number of a variety

This talk will discuss an elementary way to bound the cohomological dimensions (both coherent and topological) of a scheme X, as well as relations of this method to other numbers of interest for X. The application intended is to bound the cohomological dimension of the moduli space of curves, and we will also briefly discuss the idea in this case.

This is work in progress.

1:00pm Lunch

2:30pm: Ramesh Sreekanten (University of Toronto)

Non-Archimedean regulator maps and special values of L-functions

There are several classical formulae for special values if zeta functions - for example, $\zeta(2) = \pi^2/6$. Beilinson, unifying the work of several people over the last century, formulated conjectures which attempted to explain the special values of L-functions of algebraic varieties defined over number fields in terms of algebraic invariants of the variety. Earlier this year we talked about a function field analogue of a theorem of Beilinson's which was a special case of his conjectures - though the general conjectures in the function field setting are yet to be formulated.

In this talk we will discuss a possible formulation of general Belinson conjectures for varieties over function fields for which our earlier work provides evidence.

3:30pm: Slava Archava (McMaster University)

On the Hodge cycles of Milnor fibers

In this talk I will describe our joint project with Hossein Movasati in which we attempt to study the space of Hodge cycles on a Milnor fiber of a non-composite polynomial (and more generally on an affine hypersurface complement) using the characterization of Hodge cycles by vanishing of appropriate periods. To carry out this program we need a description (as explicit as possible) of the mixed Hodge structure on the cohomology of the variety under investigation and an explicit basis for its homology. In the case of a quasi-homogeneous polynomial we use Steenbrink's description of the Hodge filtration on the cohomology of the Milnor fiber as order of the pole filtration, generalizing classical results of Griffiths for the cohomology of a smooth hypersurface in projective space.

4:30pm: John Scherk (University of Toronto)

Borel–Serre compactifications of the classifying spaces of Hodge structures

Borel and Ji showed recently that the (reductive) Borel–Serre compactification of a hermitian symmetric space G/K can be generalized to homogeneous spaces G/K where K is any compact subgroup. In particular, this gives compactifications of classifying spaces of Hodge structures. A one-dimensional variation of Hodge structures gives a holomorphic map of a disc into such a compatification. Because of Griffiths transversality, the image can only meet certain boundary components. These will be described in terms of the root system of G and the induced Hodge structure on its Lie algebra.

6:30pm: Workshop Dinner

November 9, 2003

10:00pm: Ragnar Buchweitz (University of Toronto)

Semiregularity, Hochschild Cohomology, and the Centre of the Derived Category

Given an (algebraic or analytic) space X and a perfect complex M on it, the semiregularity map relates the deformation theory of X with that of M and its cycle class. In joint work with Flenner we show how this setup can be incorporated into a larger picture, where the Kodaira-Spencer group of first order deformations of X is replaced by Hochschild cohomology and the selfextensions of M are replaced by the graded centre of the category of perfect complexes.The theory of Grothendieck residues, as interpreted by Lipman, implies then that for a smooth affine scheme the homomorphism from Hochschild cohomology to the graded centre of the derived category is injective. We will discuss what little is known about the nature of this homomorphism in general.

11:00am Pramathanath Sastry (University of Toronto)

Abstract and concrete aspects of Grothendieck duality

The generalization of Serre Duality due to Grothendieck is far reaching and conceptually liberating. Its major drawback is that the passage from the abstract to the concrete (or vice-versa) is not so straightforward. Indeed questions like "what is the residue? " usually have two or more answers depending on which part of the concrete-abstract spectrum one is at. I will aim to give a coherent account, following the idea in Deligne's appendix to Hartshorne's book, and indicate how one bridges the gap between concrete and abstract following up on Verdier's tantalizing approach for smooth maps.